AP Calculus AB
Summer Assignment

Welcome upcoming AP Calculus students!

About the AP Calculus AB

*Building enduring mathematical understanding requires students to understand the **WHY** and **HOW** of mathematics in addition to mastering the necessary procedures and skills. To foster this deeper level of learning, AP Calculus is designed to develop mathematical knowledge conceptually, guiding students to connect topics and representations throughout each course and to apply strategies and techniques to accurately solve diverse types of problems.*

College Course Equivalents

AP Calculus AB is equivalent to a first semester college calculus course devoted to topics in differential and integral calculus.

Prerequisites

Before studying calculus, you must be familiar with elementary functions: linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric and piecewise – defined functions. You must also understand the language of functions (domain, range, odd and even, periodic, symmetry, zeros, intercepts and descriptors such as increasing or decreasing). As a prospective calculus student you should also know how the sine and cosine functions are defined from the unit circle and know the values of the trigonometric functions at the numbers: 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$ and their multiples.

This Summer Review Packet contains many of the prerequisite skills from Algebra and Pre- Calculus which we will use in calculus and with which you should be familiar. You have seen and mastered these topics in previous classes, but I understand some skills may be rusty. Take the time to complete the problems in this packet, reviewing where necessary and come prepared at the beginning of next school year. These prerequisite skills will make your time in AP Calculus much easier and greatly increase the likelihood of a successful semester.

Make sure to check your answers. Do not despair if you have difficulty with some of the problems. We will spend a few days reviewing these skills.

Please bring your completed packet the first day of school and be ready to ask questions about anything you did not understand.

Have a great summer!

SKILLS NEEDED FOR CALCULUS

I. Algebra:

*A. Exponents (operations with integer, fractional, and negative exponents)
*B. Factoring (GCF, trinomials, difference of squares and cubes, sum of cubes, grouping)
C. Rationalizing (numerator and denominator)
*D. Simplifying rational expressions
*E. Solving algebraic equations and inequalities (linear, quadratic, higher order using synthetic division, rational, radical, and absolute value equations)
F. Simultaneous equations

II. Graphing and Functions

*A. Lines (intercepts, slopes, write equations using point-slope and slope intercept, parallel, perpendicular, distance and midpoint formulas)
B. Conic Sections (circle)
*C. Functions (definition, notation, domain, range, inverse, composition)
*D. Basic shapes and transformations of the following functions (absolute value, rational, root, higher order curves, log, ln, exponential, trigonometric. piece-wise, inverse functions)
E. Tests for symmetry: odd, even

III. Geometry

A. Pythagorean Theorem
B. Area Formulas (Circle, polygons, surface area of solids)
C. Volume formulas
D. Similar Triangles

** IV. Logarithmic and Exponential Functions*

*A. Simplify Expressions (Use laws of logarithms and exponents)
*B. Solve exponential and logarithmic equations (include ln as well as log)
*C. Sketch graphs
*D. Inverses

** V. Trigonometry*

**A. Unit Circle (definition of functions, angles in radians and degrees)
B. Use of Pythagorean Identities and formulas to simplify expressions and prove identities
*C. Solve equations
*D. Inverse Trigonometric functions
E. Right triangle trigonometry
*F. Graphs

VI. Limits ?

A. Concept of a limit
B. Find limits as x approaches a number and as x approaches ∞

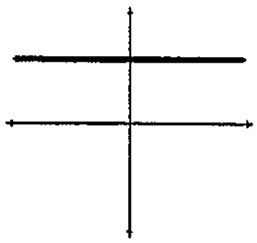
* A solid working foundation in these areas is very important.

Toolkit of Functions

Students should know the basic shape of these functions and be able to graph their transformations without the assistance of a calculator.

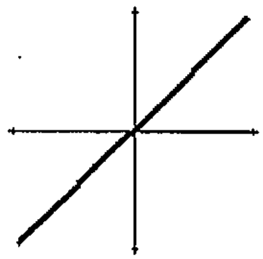
Constant

$f(x) = a$



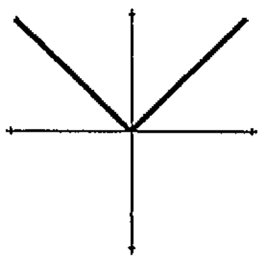
Identity

$f(x) = x$



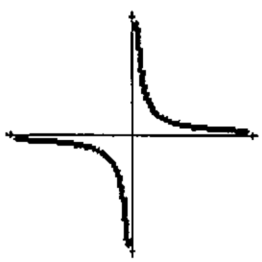
Absolute Value

$f(x) = | x |$



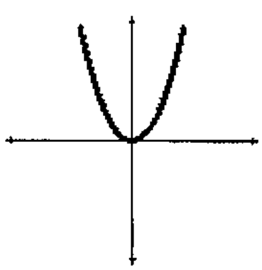
Reciprocal

$f(x) = \frac{1}{x}$



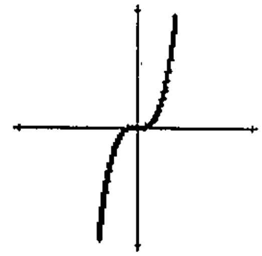
Quadratic

$f(x) = x^2$



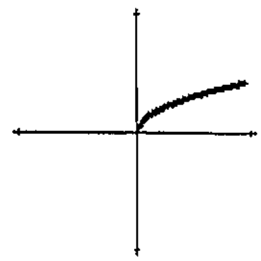
Cubic

$f(x) = x^3$



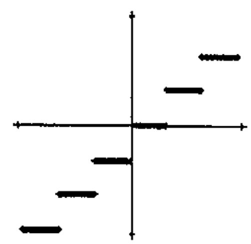
Square Root

$f(x) = \sqrt{x}$



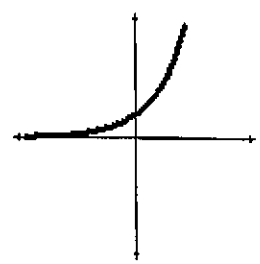
Greatest Integer

$f(x) = [x]$



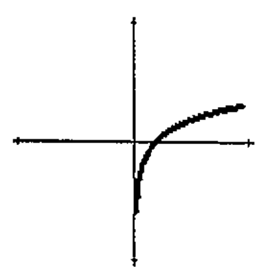
Exponential

$f(x) = a^x$



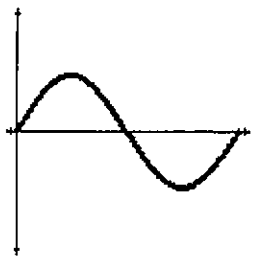
Logarithmic

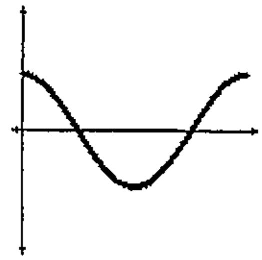
$f(x) = \ln x$

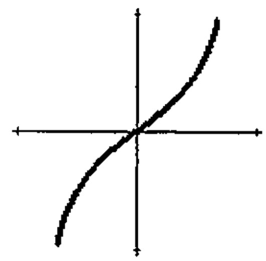


Trig Functions

$f(x) = \sin x$ $f(x) = \cos x$ $f(x) = \tan x$







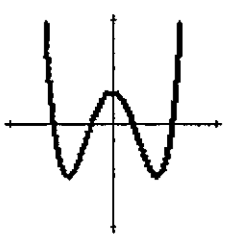
Polynomial Functions:

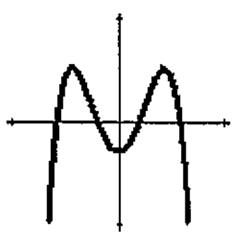
A function P is called a polynomial if $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$
Where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \ldots a_n$ are constants.

Even degree

Leading coefficient sign

Positive Negative

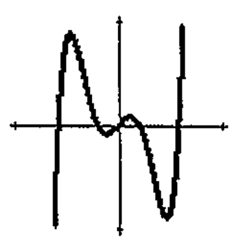


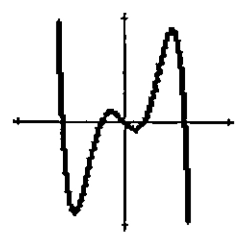


Odd degree

Leading coefficient sign

Positive Negative





- Number of roots equals the degree of the polynomial.
- Number of x intercepts is less than or equal to the degree.
- Number of "bends" is less than or equal to (degree - 1).

Trig Formulas:

Arc Length of a circle: $L = r\theta$ or $L = \frac{d}{360} \bullet 2\pi r$

Area of a sector of a circle: Area $= \frac{1}{2} r^2 \theta$ or Area $= \frac{d}{360} \bullet \pi r^2$

Solving parts of a triangle:

Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Area of a Triangle:

Area $= \frac{1}{2} bc \sin A$ or Area $= \frac{1}{2} ac \sin B$ or Area $= \frac{1}{2} ab \sin C$

Hero's formula : Area $= \sqrt{s(s-a)(s-b)(s-c)}$, where s = semi perimeter

Ambiguous Case:

θ is acute

Compute: $alt = adj \bullet \sin\theta$

opp < alt	No triangle
opp = alt	1 triangle (right)
opp>adj	1 triangle

alt<opp<adj 2 triangles

θ is obtuse or right

opp ≤ adj	No triangle
opp > adj	1 triangle

Does a triangle exist? Yes - when

(difference of 2 sides) < (third side) < (Sum of 2 sides)

Trig Identities:

Reciprocal Identities:

$\csc A = \frac{1}{\sin A}$ $\qquad$ $\sec A = \frac{1}{\cos A}$ $\qquad$ $\cot A = \frac{1}{\tan A}$

Quotient Identities: $\tan A = \frac{\sin A}{\cos A}$ $\qquad$ $\cot A = \frac{\cos A}{\sin A}$

Pythagorean Identities:

$\sin^2 A + \cos^2 A = 1$ $\quad$ $\tan^2 A + 1 = \sec^2 A$ $\quad$ $1 + \cot^2 A = \csc^2 A$

Sum and Difference Identities:

$\sin(A + B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Double Angle Identities:

$\sin(2A) = 2\sin A \cos A$

$\cos(2A) = \cos^2 A - \sin^2 A$ $\qquad$ $\cos(2A) = 2\cos^2 A - 1$ $\quad$ $\cos(2A) = 1 - 2\sin^2 A$

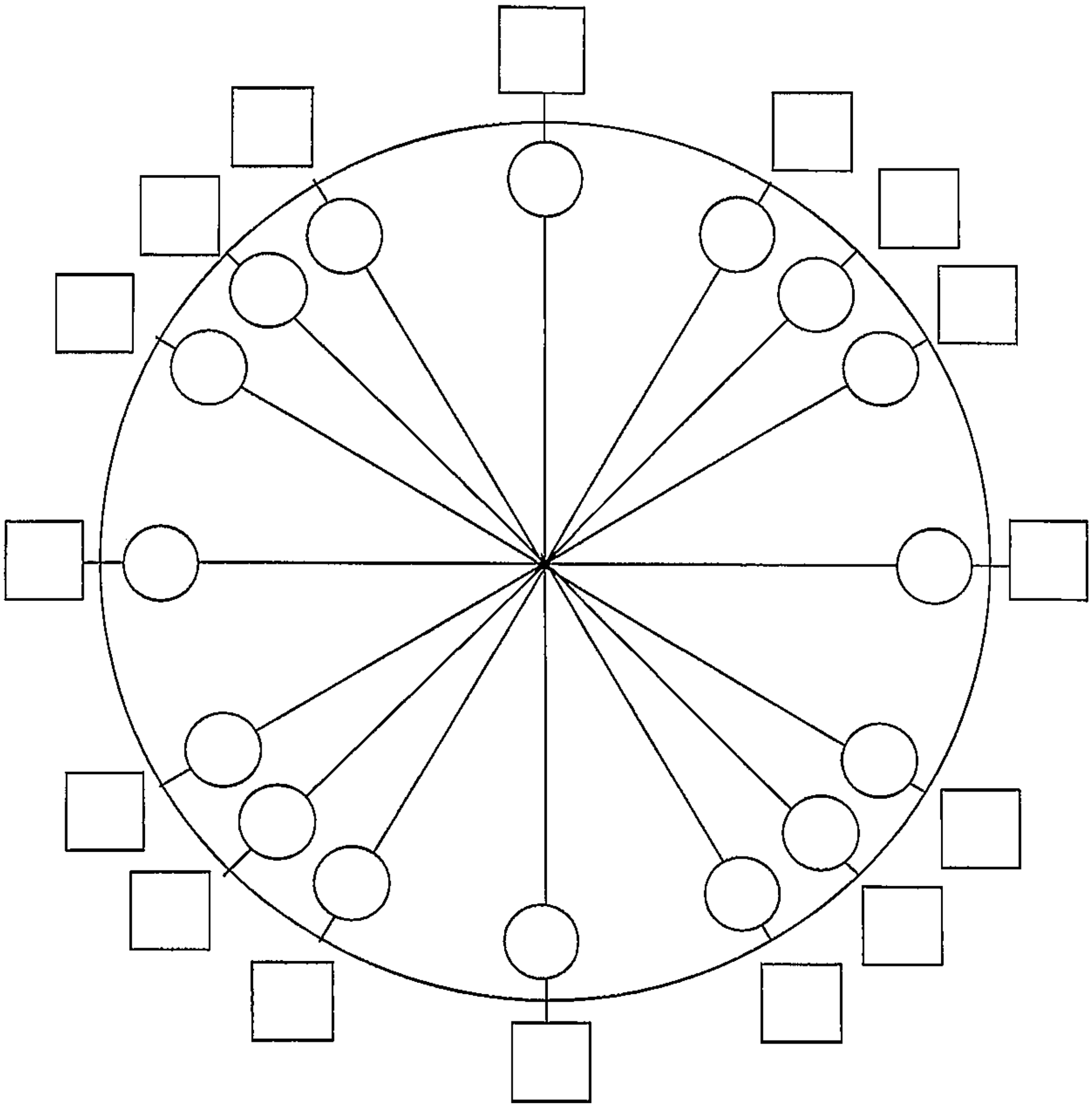
Geometric Formulas:

Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$ $\qquad$ Area of a triangle: $A = \frac{1}{2}bh$

Area of an equilateral triangle: $A = \frac{\sqrt{3}}{4}s^2$

Area if a circle: $A = \pi r^2$ $\quad$ Circumference of a circle: $C = 2\pi r$ or $C = d\pi$

Unit Circle – Degrees and Radians



Place degree measures in the circles.

Place radian measure in the squares.

Place ($\cos\theta$, $\sin\theta$) in parenthesis outside the square.

Place $\tan\theta$ outside the parenthesis.

$\tan\theta =$ ______________

$\cot\theta =$ ______________

$\csc\theta =$ ______________

$\sec\theta =$ ______________

Calculus Prerequisite Problems

Work the following problems on your own paper. Show all necessary work.

I. Algebra

A. Exponents: 1) $\dfrac{(8x^3yz)^{1/3}(2x)^3}{4x^{1/3}\left(yz^{2/3}\right)^{-1}}$

B. Factor Completely:

2) $9x^2 + 3x - 3xy - y$ (use grouping) 3) $64x^6 - 1$ *Hint: Factor as difference of squares first, then as the sum and difference of cubes second.*

4) $42x^4 + 35x^2 - 28$ 5) $15x^{5/2} - 2x^{3/2} - 24x^{1/2}$ Hint: Factor GCF $x^{1/2}$ first.

6) $x^{-1} - 3x^{-2} + 2x^{-3}$ Hint: Factor out GCF x^{-3} first.

C. Rationalize denominator / numerator:

7) $\dfrac{3-x}{1-\sqrt{x-2}}$ 8) $\dfrac{\sqrt{x+1}+1}{x}$

D. Simplify the rational expression:

9) $\dfrac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

E. Solve algebraic equations and inequalities

10. – 11. Use synthetic division to help factor the following, state all factors and roots.

10) $p(x) = x^3 + 4x^2 + x - 6$ 11) $p(x) = 6x^3 - 17x^2 - 16x + 7$

12) <u>*Explain*</u> why $\frac{3}{2}$ cannot be a root of $f(x) = 4x^5 + cx^3 - dx + 5$, where c and d are integers. (hint: You can look at the possible rational roots.)

13) <u>*Explain*</u> why $f(x) = x^4 + 7x^2 + x - 5$ must have a root in the interval $[0, 1]$, ($0 \le x \le 1$) Check the graph and use signs of f(0) and f(1) to justify you're answer.

Solve: You may use your graphing calculator to check solutions.

14) $(x+3)^2 > 4$ 15) $\dfrac{x+5}{x-3} \le 0$ 16) $3x^3 - 14x^2 - 5x \le 0$ (Factor first)

17) $x < \dfrac{1}{x}$ 18) $\dfrac{x^2-9}{x+1} \ge 0$ 19) $\dfrac{1}{x-1} + \dfrac{4}{x-6} > 0$

20) $x^2 < 4$ 21) $|2x+1| < \dfrac{1}{4}$

F. Solve the system. Solve the system algebraically and then check the solution by graphing each function and using your calculator to find the points of intersection.

22) $x - y + 1 = 0$
$y - x^2 = -5$

23) $x^2 - 4x + 3 = y$
$-x^2 + 6x - 9 = y$

II. Graphing and Functions:

A. Linear graphs: Write the equation of the line described below.

24) Passes through the point (2, -1) and has slope $-\frac{1}{3}$.

25) Passes through the point (4, - 3) and is perpendicular to $3x + 2y = 4$.

26) Passes through (-1, - 2) and is parallel to $y = \frac{3}{5}x - 1$.

B. Conic Sections: Write the equation in standard form and identify the conic.

28) $4x^2 - 16x + 4y^2 + 24y + 52 = 0$

C. Functions: Find the domain and range of the following.

Note: domain restrictions - denominator $\neq 0$, argument of a log or ln > 0, radicand of even index must be ≥ 0
range restrictions- reasoning, if all else fails, use graphing calculator

29) $y = \frac{3}{x-2}$

30) $y = \log(x - 3)$

31) $y = x^4 + x^2 + 2$

32) $y = \sqrt{2x - 3}$

33) $y = | x - 5 |$

34) domain only: $y = \frac{\sqrt{x+1}}{x^2 - 1}$

35) Given f(x) below, graph over the domain [-3, 3], what is the range?

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 1 & \text{if } -1 \leq x < 0 \\ x - 2 & \text{if } x < -1 \end{cases}$$

Find the composition /inverses as indicated below.

Let $f(x) = x^2 + 3x - 2$ $\quad g(x) = 4x - 3$ $\quad h(x) = \ln x$ $\quad w(x) = \sqrt{x-4}$

36) $g^{-1}(x)$ 37) $h^{-1}(x)$ 38) $w^{-1}(x)$, for $x \geq 4$ 39) $f(g(x))$ 40) $h(g(f(1)))$

41) Does $y = 3x^2 - 9$ have an inverse function? Explain your answer.

Let $f(x) = 2x$, $g(x) = -x$, and $h(x) = 4$, find

42) $(f \circ g)(x)$ 43) $(f \circ g \circ h)(x)$

44) Let $s(x) = \sqrt{4-x}$ and $t(x) = x^2$, find the domain and range of $(s \circ t)(x)$.

D. *Basic Shapes of Curves:*

Sketch the graphs. You may use your graphing calculator to verify your graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting a few points, and by your knowledge of transformations.

45) $y = \sqrt{x}$ 46) $y = \ln x$ 47) $y = \frac{1}{x}$ 48) $y = |x - 2|$

49) $y = \frac{1}{x-2}$ 50) $y = \frac{x}{x^2-4}$ 51) $y = 2^{-x}$ 52) $y = 3\sin 2\left(x - \frac{\pi}{6}\right)$

53) $f(x) = \begin{cases} \sqrt{25-x^2} & \text{if } x < 0 \\ \dfrac{x^2-25}{x-5} & \text{if } x \geq 0, x \neq 5 \\ 0 & \text{if } x = 5 \end{cases}$

E. *Even, Odd, Tests for Symmetry:*

Identify as odd, even , or neither and justify you're answer. *<u>To justify your answer you must show substitution using -x ! It is not enough to simply check a number.</u>*

Even: $f(x) = f(-x)$ $\quad$ Odd: $f(-x) = -f(x)$

54) $f(x) = x^3 + 3x$ 55) $f(x) = x^4 - 6x^2 + 3$ 56) $f(x) = \frac{x^3 - x}{x^2}$

57) $f(x) = \sin 2x$ 58) $f(x) = x^2 + x$ 59) $f(x) = x(x^2 - 1)$

60) $f(x) = \frac{1 + |x|}{x^2}$

61) What type of function (even or odd) results from the product of two

even functions? odd functions?

Test for symmetry. Show substitution with variables to justify you're answer.

Symmetric to y axis: replace x with - x and relation remains the same.
Symmetric to x axis: replace y with - y and relation remains the same.
Origin symmetry: replace x with - x, y with - y and the relation is equivalent.

62) $y = x^4 + x^2$ 63) $y = \sin(x)$ 64) $y = \cos(x)$

65) $x = y^2 + 1$ 66) $y = \dfrac{|x|}{x^2 + 1}$

IV LOGARITHMIC AND EXPONENTIAL FUNCTIONS

A. Simplify Expressions:

67) $\log_4\left(\frac{1}{16}\right)$ 68) $3\log_3 3 - \frac{3}{4}\log_3 81 + \frac{1}{3}\log_3\left(\frac{1}{27}\right)$ 69) $\log_9 27$

70) $\log_{125}\left(\frac{1}{5}\right)$ 71) $\log_w w^{45}$ 72) $\ln e$ 73) $\ln 1$ 74) $\ln e^2$

B. Solve equations:

75) $\log_6(x+3) + \log_6(x+4) = 1$ 76) $\log x^2 - \log 100 = \log 1$ 77) $3^{x+1} = 15$

V TRIGONOMETRY

A. Unit Circle: Know the unit circle – radian and degree measure. Be prepared for a quiz.

78) State the domain, range and fundamental period for each function?

a) $y = \sin x$ b) $y = \cos x$ c) $y = \tan x$

B. Identities:

Simplify: 79) $\dfrac{(\tan^2 x)(\csc^2 x) - 1}{(\csc x)(\tan^2 x)(\sin x)}$ 80) $1 - \cos^2 x$ 81) $\sec^2 x - \tan^2 x$

82) Verify: $(1 - \sin^2 x)(1 + \tan^2 x) = 1$

C. *Solve the Equations*

83) $\cos^2 x = \cos x + 2, \quad 0 \le x \le 2\pi$ 84) $2\sin(2x) = \sqrt{3}, \quad 0 \le x \le 2\pi$

85) $\cos^2 x + \sin x + 1 = 0, \quad 0 \le x \le 2\pi$

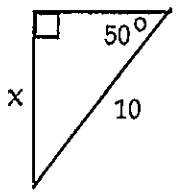
D. Inverse Trig Functions: *Note: $Sin^{-1}x$ = Arcsin x*

86) Arcsin1 87) $\text{Arcsin}\left(-\frac{\sqrt{2}}{2}\right)$ 88) $\text{Arccos}\left(\frac{\sqrt{3}}{2}\right)$ 89) $\sin\left(\text{Arccos}\left(\frac{\sqrt{3}}{2}\right)\right)$

90. State domain and range for: Arcsin(x) , Arccos (x), Arctan (x)

E. Right Triangle Trig: Find the value of x. (Note: Degree measure!)

91.



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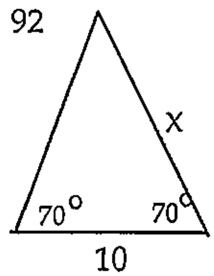

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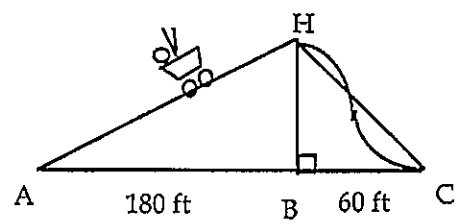


93) The roller coaster car shown in the diagram above takes 23.5 sec. to go up the 23 degree incline segment AH and only 2.8 seconds to go down the drop from H to C. The car covers horizontal distances of 180 feet on the incline and 60 feet on the drop. Decimals in answer may vary.
  a. How high is the roller coaster above point B?
  b. Find the distances AH and HC.
  c. How fast (in ft/sec) does the car go up the incline?
  d. What is the approximate average speed of the car as it goes down the drop?

F. Graphs: Identify the amplitude, period, horizontal, and vertical shifts of these functions.

94) $y = -2\sin(2x)$ 95) $y = -\pi\cos\left(\frac{\pi}{2}x + \pi\right)$

102. The number N (in thousands) of cellular phone subscribers in Malaysia is shown in the table. (Midyear estimates are given.)

t	1991	1993	1995	1997
N	132	304	873	2461

(a) Use the data to sketch a rough graph of N as a function of t.
(b) Use your graph to estimate the number of cell-phone subscribers in Malaysia at midyear in 1994 and 1996.

103. If $f(x) = 3x^2 - x + 2$, find f(2), f(-2), f(a), f(-a), f(a+1), 2f(a), $f(a^2)$, $[f(a)]^2$, and f(a+h).

104. Find the domain of each function.

a) $f(x) = \frac{x}{3x-1}$

b) $g(u) = \sqrt{u} + \sqrt{4-u}$

105. Find an expression for the bottom half of the equation $x + (y-1)^2 = 0$.

106. A rectangle has perimeter 20 m. Express the area of the rectangle as a function of the length of one of its sides.

108. Biologists have noticed that the chirping rate of crickets of a certain species is related to temperature, and the relationship appears to be very nearly linear. A cricket produces 113 chirps per minute at 70° F and 173 chirps per minute at 80° F.

(a) Find a linear equation that models the temperature T as a function of the number of chirps per minute N.

(b) What is the slope of the graph? What does it represent?

(c) If the crickets are chirping at 150 chirps per minute, estimate the temperature.

109. At the surface of the ocean, the water pressure is the same as the air pressure above the water, 15 lb/in^2. Below the surface, the water pressure increases by 4.34 lb/in^2 for every 10 ft of descent.

(a) Express the water pressure as a function of the depth below the ocean surface.

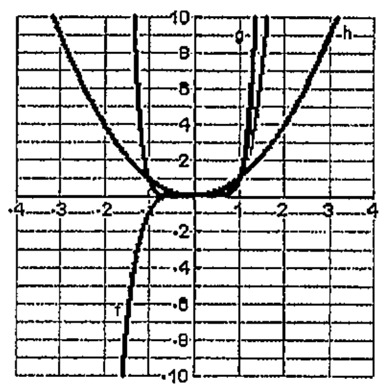
(b) At what depth is the pressure 100 lb/in^2?

110. Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, or logarithmic function.

(a) $f(x)=\sqrt[5]{x}$ (b) $g(x)=\sqrt{1-x^2}$ (c) $h(x)=x^9+x^4$ (d) $r(x)=\dfrac{x^2+1}{x^3+x}$

111. Match each equation with its graph. Explain your choices. (Don't use a computer or graphing calculator).

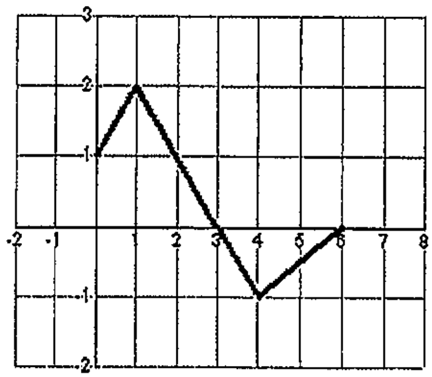
(a) $y=x^2$ (b) $y=x^5$ (c) $y=x^8$



112. Suppose the graph of f is given. Write equations for the graphs that are obtained from the graph of f as follows.

(a) Shift 3 units upward.
(b) Shift 3 units downward.
(c) Shift 3 units to the right.
(d) Shift 3 units to the left.
(e) Reflect about the x-axis.
(f) Reflect about the y-axis.
(g) Stretch vertically by a factor of 3.
(h) Shrink vertically by a factor of 3.

114. The graph of f is given. Use it to graph the following functions.
(a) y = f(2x)   (b)=f(½ x)   (c) y = f(-x)   (d) y = -f(-x)



115. Graph the following, not by plotting points, but by starting with the graph of one of the standard functions and applying the appropriate transformations.

$$y = \frac{1}{3}\sin\left(x - \frac{\pi}{6}\right)$$

116. Find f+g, f - g, fg, and f/g and state their domains.

$f(x) = x^3 + 2x^2, \qquad g(x) = 3x^2 - 1$

117. Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.

$f(x) = \sin x, \qquad g(x) = 1 - \sqrt{x}$

118. Express the function in the form $f \circ g$.

$F(x) = (x^2 + 1)^{10}$

Part A (Easy problems and problems of moderate difficulties)

1) *For each y = f(x) find domain, range, x – intercept, y – intercept and sketch the graph (do not use your graphic calculator)*

a) $y = 2^x$

b) $y = 1 + 3^{-2x}$

c) $y = 4^{1-x}$

d) $y = \left(\frac{1}{5}\right)^{x-4}$

e) $y = \left(\frac{1}{2}\right)^{-3x+1}$

f) $y = log_2 x$

g) $y = \ln\frac{1}{x}$

h) $y = log_{\frac{1}{2}}(x+2)$

i) $y = log_3\frac{x-1}{9}$

j) $y = log_{0.5}(8x)$

2) *Solve the equation*

a) $log_5(x-3) = log_5\sqrt{x+3}$

b) $e^{\ln(1+x)} = (2x+2)^2$

c) $log_2 x - log_2(\sqrt{x} - 1) = 2$

d) $e^{\ln(1-x)^2} = x^2 - 4x + 3$

e) $4^x + 2^x - 2 = 0$

f) $\left(\frac{1}{2}\right)^{2x} - \left(\frac{1}{2}\right)^x - 6 = 0$

g) $log_3^2 x + log_3 x - 2 = 0$

3) *Solve each inequality (do not use your graphing calculator)*

a) $4^x < 8$

b) $3^x < \frac{1}{9}$

c) $2 \cdot 2^{-x} < 4$

d) $9^{x-2} > \frac{1}{3}$

e) $2 \cdot 4^{x+1} < \frac{1}{8}$

f) $log_2 x < 1$

g) $log_{\frac{1}{2}}(x+2) > 4$

h) $\log(x-3) < -2$

i) $\ln\left(\frac{1}{x}\right) > 3$

j) $log_2\left(log_{\frac{1}{3}} x\right) < 1$

k) $x^2 - 3x + 2 < 0$

l) $x^2 - 3x + 2 \geq 0$

m) $4^x - 3 \cdot 2^x + 2 < 0$

n) $log_2^2 x - 3log_2 x + 2 < 0$

4) *Find the domain for each given function:*

a) $f(x) = log_3(x-1)$

b) $f(x) = \log(x^2+3)$

c) $f(x) = log_2(4-8x)$

d) $f(x) = \frac{2x+1}{x^2-1} + \frac{7}{\sqrt{x-1}} - \frac{3x}{x^2}$

e) $f(x) = \frac{4}{4-9x^2} + \frac{1}{x^3} - 27$

f) $f(x) = \frac{x-1}{x^2-1} - \frac{4}{\sqrt{x^2-4}}$

g) $f(x) = \frac{3x}{\sqrt{1-x}} + \frac{7-x}{\sqrt{x+6}}$

h) $f(x) = \sqrt{\frac{x^2-x}{x+1}}$

i) $f(x) = \frac{\ln(3-x)}{\sqrt{x^2+x+1}}$

5) $g(x) = x^3 + 4x$ Find a) $g(2a)$ b) $g(-3a^2)$ c) $g(3-4a)$

6) Simplify:

a) $\dfrac{x-2}{\sqrt{x+2}-2}$

b) $\dfrac{x^2-1}{x} \div \dfrac{x+1}{x^3}$

c) $\dfrac{\frac{1}{x}+\frac{4}{x^2}}{3-\frac{1}{x}}$

d) $\dfrac{1}{1-2a} - \dfrac{2}{1+2a} + \dfrac{6a+2}{4a^2-1}$

e) $\dfrac{\frac{a}{2x+h}-\frac{a}{2x}}{h}$

f) $\dfrac{3(x+h)^2-3x^2}{h}$

g) $\sqrt{x} \cdot \sqrt[3]{x} \cdot x^{\frac{1}{6}}$

7) True / False problems:

a) $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$ (T/F)

b) $\frac{1}{p+g} = \frac{1}{p} + \frac{1}{g}$ (T/F)

c) $\frac{3(a+b)}{c} = \frac{3a+b}{c}$ (T/F)

d) $\sqrt{a^2+b^2} = a + b$ (T/F)

e) $\frac{2k}{2x+h} = \frac{k}{x+h}$ (T/F)

8) $xy + y + x = y$ a) Solve for x b) Solve for y

9) Factor: a) $x^2(x-1) - 4(x-1)$ b) $x^4(2-x) - 16(2-x)$

10) Evaluate each of the following – leave answer in radical form (do not use calculator)

a) $\sin\frac{\pi}{6} - \cos\frac{2\pi}{3} =$

b) $\sin\left(-\frac{\pi}{4}\right) + \tan\pi =$

c) $\cos\frac{3\pi}{4} - \cos^{-1}\left(\frac{1}{2}\right) =$

d) $\sin^{-1}\frac{\pi}{6} + \tan^{-1}(1) =$

11) Solve:

a) $\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$ $\quad 0 \le x \le 2\pi$

b) $2\sin^2 x + \sin x - 1 = 0$ $\quad 0 \le x \le 2\pi$

12) Show work to determine if the relation is even, odd or neither:

a) $f(x) = 3x^2 - 6$

b) $f(x) = -2x^3 - 7x$

c) $f(x) = 3x^2 - 3x + 3$

13) Find the equation of a line:

a) Passing through (-2,4) and (3,-5)

b) Passing through (3,7) and parallel to the line 3x+2y-8=0

c) That is perpendicular to the line 3x+2y-8=0 at the point $\left(3, -\frac{1}{2}\right)$

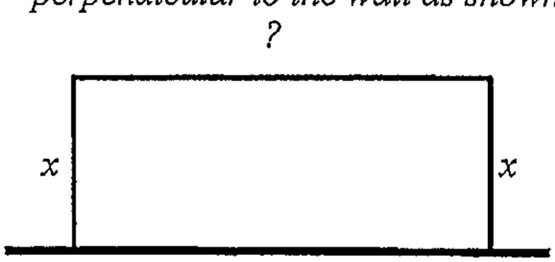
14) *The line with slope 4 that passes through (-3,7) interests the x – axis at point A and y – axis at point B. Find the coordinates of each point.*

15) $f(x) = \frac{1}{x-1}$ *and* $g(x) = x^2 - 2$
Find: a) f(g(x)), b) g(f(x)), c) f(f(x)), d) g(g(x))

16) *Find the surface area of a box of height **h** whose base dimensions are **l** and w that satisfies the following condition:*
a) *The box is closed, the base is rectangle.*
b) *The box has an open top, the base is a right triangle with legs **l** and w*
c) *The box has an open top and the base is square with side **x***

17) *For each function: find domain, find range, sketch the graph. (do not use your calculator)*
a) $y = \sqrt{x-2} + 1$
b) $y = \frac{x-2}{x^2-4}$
c) $y = |x+3| - 2$
d) $y = \sqrt{4 - x^2}$
e) $y = x^3 - x$
f) $y = x^3 + x$

18) *Three sides of a fence and an existing wall form a rectangular enclosure. The total length ef a fence used for the three sides is 240 ft. Let x be the length of two sides perpendicular to the wall as shown below.*



Existing wall

a) *Write the equation of area A of the enclosure as a function of the length **x** of the rectangular area as shown in the above figure.*
b) *Find value(s) of **x** for which the area is 5500* ft^2
c) *What is the maximum area?*

Part B (problems of moderate difficulties and challenge problems)

1) *Express* $|2x-4|$ *as a piece-wise function*

3) *Simplify : a)* $\frac{(x^2+2x)-(a^2+2a)}{x-a}$

b) $\frac{\frac{-2}{x-3}-x}{x^2-1}$

5) *Solve the equation:* $4^x - 2^x - 12 = 0$

8) $f(x) = x^2 + 3x$ *Does y = f(x) have an inverse? Explain why.*

9) $f(x) = -x^2 + 4x$ *Does y = f(x) have an inverse? Explain why.*

a) When $x < 2$

b) When $x > 0$

10) $f(x) = 2^{3-x}$

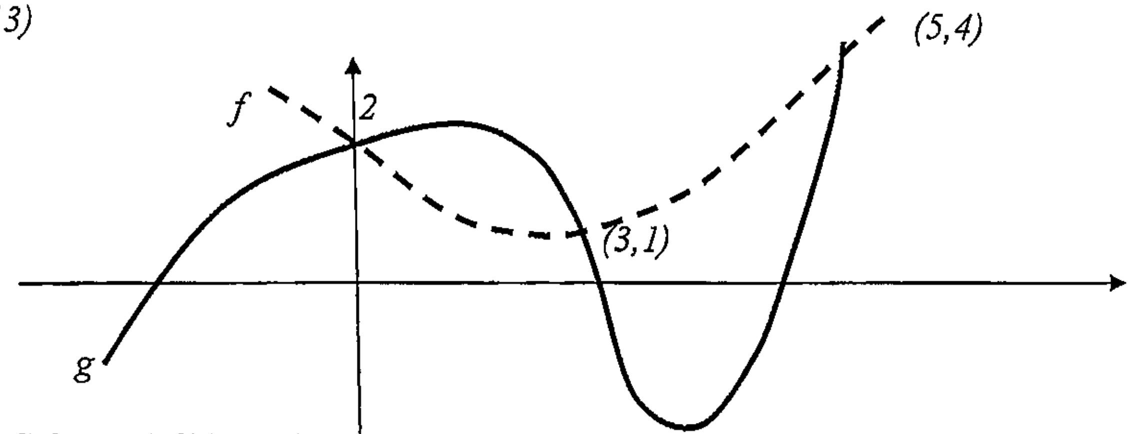
a) Sketch the graph of y = f(x)

b) Find $y = f^{-1}(x)$ *and sketch the graph.*

c) Check (both algebraic and geometrical method) that $y = f^{-1}(x)$ *is the inverse of y = f(x)*

12) $f(x) = \begin{cases} x^2 - 4 & -2 \le x \le 2 \\ x - 2 & 2 < x \le 6 \end{cases}$ *Sketch the graph, find domain, find range.*

13)



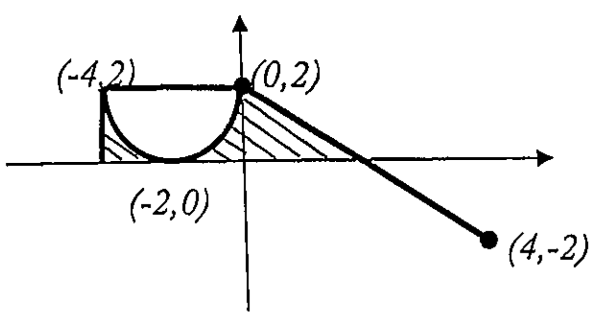
Solve a) $f(x) > g(x)$ b) $f(x) < g(x)$

14) Find m and b such that for any real value for x $2x - (mx+b) = m$

15) $-\ln(3 - y) = \sin x - \ln 2$ Find $y = f(x)$

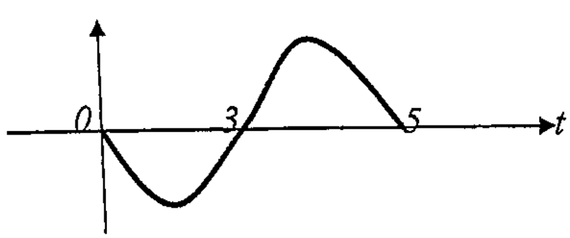
16) $f(x) = \frac{2}{\sqrt{5-4x^2}}$ Find domain.

17)



Find the area of the shaded region.

19)



The graph of $y = v(t)$ is given left

a) Solve $v(t) < 0$

b) Solve $v(t) > 0$

20) Solve: $\cos\left(\frac{\pi}{6}t\right) < 0$ for $0 \le t \le 12$

21) Sketch the graph:

a) $x^2 + y^2 = 4$

b) $y = +\sqrt{4 - x^2}$

c) $y = -\sqrt{4 - x^2}$

d) $x = -\sqrt{4 - y^2}$

e) $x = +\sqrt{4 - y^2}$

22) $f(x) = \sqrt{25 - x^2}$

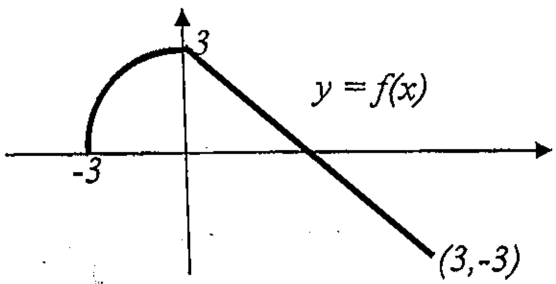
$g(x) = \begin{cases} f(x) & -5 \le x \le -3 \\ x + 7 & -3 < x \le 5 \end{cases}$

Sketch the graph and find domain and range

a) for y = f(x) *b) for y = g(x)*

23) *Express* $y = |x^2 - 16|$ *as a piece – wise function*

25)



The graph of y = f(x) is given.
Find f(x) using a piece-wise function