Inv 1.1 Testing Bridge Thickness

A. Answers will vary based on experimental data. Sample:

<table>
<thead>
<tr>
<th>Thickness (layers)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breaking Weight (pennies)</td>
<td>9</td>
<td>16</td>
<td>24</td>
<td>34</td>
<td>42</td>
</tr>
</tbody>
</table>

B. Possible graph:
C. The relationship is approximately linear. In the table, this is shown by the near-constant differences in breaking weights for consecutive thickness values. In the graph, this is shown by the near straight-line pattern of points. The relationship is also increasing. That is, as the thickness increases, the breaking weight increases.

D. Based on the previous data, one possible prediction is 20 pennies. As thickness increases by 1 layer, the breaking weight increases by about 8 pennies. So as thickness increases by half a layer, breaking weight should increase by about 4 pennies.
1. Based on the data above, one possible prediction is 50 pennies. As thickness increases by 1 layer, the breaking weight increases by about 8 pennies. Therefore, 6 layers would probably have a breaking weight of $42 + 8 = 50$ pennies.

2. Possible answer: The breaking weight is probably not actually a whole number of pennies. Also, the rate of change is not exactly 8 pennies every time, so predictions of the last value of 42 plus 8 might not match the actual breaking weight.
Inv 1.2 Testing Bridge Length

A. Answers will vary.

<table>
<thead>
<tr>
<th>Length (in.)</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>9</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breaking Weight (pennies)</td>
<td>42</td>
<td>26</td>
<td>19</td>
<td>16</td>
<td>14</td>
</tr>
</tbody>
</table>

B. Graphs will vary. This is the graph of the data in Figure 1.
C. Possible answer: As length increases, breaking weight decreases, but the relationship is not linear. In the table, the breaking weights decrease as the lengths increase, but not at a constant rate. In the graph, the pattern of points is a curve that decreases at a slower and slower rate.

D. Predictions and explanations will vary. Students may estimate using the pattern in either the graph or the table. Based on the graph in B, we might predict that the breaking weights are 58, 34, 15, and 13, respectively. The results might not match exactly because the actual breaking weights might not be whole pennies. Whole pennies might be too “coarse” a unit of measure.
E. Possible answer: They are similar in that breaking weight depends on another variable—either bridge thickness or bridge length. However, the nature of the two relationships is very different. As thickness increases, breaking weight increases. As length increases, breaking weight decreases. Furthermore, the relationship between bridge thickness and breaking weight appears to be roughly linear, while the relationship between bridge length and breaking weight does not.
Inv 1.3 Custom Construction Parts

A. 1. (Figure 4)

Figure 4

<table>
<thead>
<tr>
<th>Beam Length (ft)</th>
<th>CSP Beams</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>19</td>
</tr>
<tr>
<td>6</td>
<td>23</td>
</tr>
<tr>
<td>7</td>
<td>27</td>
</tr>
<tr>
<td>8</td>
<td>31</td>
</tr>
</tbody>
</table>

2. CSP Beams

Number of Rods

Beam Length (ft)
3. Every time another foot is added to the length, 4 rods are added.

4. In the table, each increase of 1 ft in the beam length yields an increase of 4 in the number of rods. The graph is a straight line. To get from one point to the next, you move over 1 and up 4.

5. 199 rods; explanations will vary.

B. 1. (Figure 5)

<table>
<thead>
<tr>
<th>Number of Steps</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Rods</td>
<td>4</td>
<td>10</td>
<td>18</td>
<td>28</td>
<td>40</td>
<td>54</td>
<td>70</td>
<td>88</td>
</tr>
</tbody>
</table>
5. 199 rods; explanations will vary.

B. 1. (Figure 5)

2. CSP Staircase Frames

<table>
<thead>
<tr>
<th>Number of Steps</th>
<th>Number of Rods</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>
3. Possible answer: As you increase the number of steps by 1, the number of rods increases by the next even number.

4. In the table, the number of rods increases by 6, 8, 10, and so on. The graph curves upward at an increasing rate.

5. 180 rods

C. Both patterns are increasing. The beam relationship is linear, so it has a straight-line graph and a table with constant differences. The staircase relationship is nonlinear, so it has a curved graph and a table in which the differences are nonconstant. Both graphs increase from left to right.

D. The beam and bridge-thickness relationships are both increasing and linear. The bridge-length and staircase relationships are both nonlinear, but the former is increasing and the latter is decreasing.
Inv 2.1 Linear Models

First State Bridge-Painting Costs

<table>
<thead>
<tr>
<th>Bridge Number</th>
<th>Length (ft)</th>
<th>Painting Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>$18,000</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>$37,000</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>$48,000</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
<td>$66,000</td>
</tr>
</tbody>
</table>

The First State cost estimators plot the data. The points fall in a nearly linear pattern. They draw a line that fits the pattern well. The line is a mathematical model for the relationship between bridge length and painting cost. A mathematical model approximates a data pattern.
A. 1. Write an equation for the line that models the data.

A. 1. \( y = 150x + 5000 \). Using coordinates from the line, such as \((100, 20,000)\) and \((300, 50,000)\), the slope is the ratio of the vertical change to the horizontal change:
\[
\frac{30,000}{200} = 150.
\]
The \(y\)-intercept can be estimated from the graph as \((0, 5,000)\).

2. Use the line or the equation to estimate painting costs for similar bridges that are
   a. 175 feet long
   b. 280 feet long

2. a. \$31,250. Use the equation,
   \[ y = 150x + 5,000, \]
   to get
   \[
   150 \cdot 175 + 5000 = 31,250.
   \]
   Or, estimate by looking on the line for the \(y\)-coordinate corresponding to an \(x\)-coordinate of 175.

b. \$47,000. Solution methods are similar to those described in part (a).
3. Use the line or the equation to estimate lengths of similar bridges for which the painting costs are
   a. $10,000  
   b. $60,000

3. a. About 33 ft. Find the x-coordinate of the point that has a y-coordinate of 10,000. Or, to find the exact value, solve $10,000 = 150x + 5,000$. The solution is $x = 33\frac{1}{3}$ ft.

   b. About 370 ft. Solution methods are similar to those described in part (a).

B. First State is also bidding on a different type of bridge. It has records for three similar bridges.

<table>
<thead>
<tr>
<th>First State Bridge-Painting Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bridge Number</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>

1. Plot these data points. Draw a line that models the pattern in the data points.
B. 1. Possible line:

2. Write an equation for your line.

2. Possible answer: \( y = 250x + 8,000 \)
3. Use your equation or line to estimate the painting cost for a similar bridge that is 200 feet long.

3. Answer should be close to $60,000. Students should either use the line to estimate the y-coordinate of the point with x-coordinate 200 or calculate $250(200) + 8,000$.

4. Use your equation or line to estimate the length of a similar bridge that costs $100,000 to paint.

4. Answer should be close to 370 ft. Students may use the line to estimate the x-coordinate of the point with y-coordinate 100,000 or solve the equation $100,000 = 250x + 8,000$. 
Inv 2.2 Equations for Linear Relationships

Getting Ready

1. Sudzo Wash and Wax charges customers $0.75 per minute to wash a car. Write an equation that relates the total charge $c$ to the amount of time $t$ in minutes.

   \[ c = 0.75t \]

2. Pat’s Power Wash charges $2.00 per car to cover the cost of cleaning supplies, plus $0.49 per minute for the use of water sprayers and vacuums. Write an equation for the total charge $c$ for any car-wash time $t$.

   \[ c = 0.49t + 2.00 \]

3. U-Wash-It charges $10 for each car. The business owners estimate that it costs them $0.60 per minute to provide soap, water, and vacuums for a car. Write an equation for the profit $p$ U-Wash-It earns if a customer spends $t$ minutes washing a car.

   \[ p = 10.00 - 0.60t \]

Explain what the numbers and variables in each equation represent.

What questions can your equations help you answer?
A. The Squeaky Clean Car Wash charges by the minute. This table shows the charges for several different times.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge</td>
<td>$8</td>
<td>$13</td>
<td>$18</td>
<td>$23</td>
<td>$28</td>
</tr>
</tbody>
</table>

1. Explain how you know the relationship is linear.
2. What are the slope and y-intercept of the line that represents the data?
3. Write an equation relating charge $c$ to time $t$ in minutes.

A. 1. For each 5-minute change in time, the charge increases by $5. Because the rate of change is constant, the relationship is linear.
2. The slope is 1 and the y-intercept is 3.
3. $c = t + 3$
B. Euclid’s Car Wash displays its charges as a graph. Write an equation for the charge plan at Euclid’s. Describe what the variables and numbers in your equation tell you about the situation.

\[ c = \frac{1}{3}t + 4 \]

where \( t \) is time used in minutes and \( c \) is the charge in dollars. The slope is \( \frac{1}{3} \). It tells you the increase in charge for each additional minute of time. The 4 is the \( y \)-intercept, or starting value. It is a fixed charge customers pay.
C. Below are two receipts from Super Clean Car Wash. Assume the relationship between charge $c$ and time used $t$ is linear.

<table>
<thead>
<tr>
<th>SUPER CLEAN Car Wash</th>
<th>SUPER CLEAN Car Wash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date: 3-14-05</td>
<td>Date: 4-04-05</td>
</tr>
<tr>
<td>Start time: 01:55 pm</td>
<td>Start time: 09:30 am</td>
</tr>
<tr>
<td>Stop time: 02:05 pm</td>
<td>Stop time: 09:50 am</td>
</tr>
<tr>
<td>Charge: $7.00</td>
<td>Charge: $12.00</td>
</tr>
</tbody>
</table>

1. Each receipt represents a point ($t, c$) on the line. Find the coordinates of the two points.
2. What are the slope and $y$-intercept of the line?
3. Write an equation relating $c$ and $t$.

C. 1. (10, 7) and (20, 12)
   
   2. The slope is $\frac{1}{2}$, and the $y$-intercept is 2.
   
   3. $c = \frac{1}{2}t + 2$

D. Write an equation for the line with slope $-3$ that passes through the point (4, 3).

D. $y = -3x + 15$
E. Write an equation for the line with points (4, 5) and (6, 9).

\[ y = 2x - 3 \]

F. Suppose you want to write an equation of the form \( y = mx + b \) to represent a linear relationship. What is your strategy if you are given

1. a description of the relationship in words?
2. two or more \((x, y)\) values or a table of \((x, y)\) values?
3. a graph showing points with coordinates?

F. 1. Possible answer: Decide what the independent and dependent variables are. To find the value of \( m \), look for a rate of increase or decrease. Sometimes the \( b \) value is given in words as a starting value. If it is not, use the rate of change to work backward to find what the value of the dependent variable would be when the independent variable is 0.
2. Possible answer: Find how much the $y$-value changes for every increase of 1 in the $x$-value. This rate of change can be determined by finding the ratio of the difference in two $y$-values to the difference in the corresponding two $x$-values, or

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$ The $y$-intercept can be found by working backward from given points to the point $(0, \_\_)$ using slope, or rate of change, information. Or, once $m$ is known, substitute given values of $x$ and $y$ into the equation $y = mx + b$ and solve to find $b$.

3. Possible answer: Find the coordinates of two points the line passes through and proceed as described in the answer to part (2) to find the slope. Look for the coordinates of the $y$-intercept to find $b$. 
Inv 2.3 Solving Linear Equations

One step equations:

\[ 9 = b + 4 \]
\[ -4 = h + 3 \]
\[ 5 = x - 2 \]
\[ \begin{align*}
 9 - 4 &= b + 4 - 4 \\
 5 &= b
\end{align*} \]
\[ \begin{align*}
 -4 - 3 &= h + 3 - 3 \\
 -7 &= h
\end{align*} \]
\[ \begin{align*}
 5 + 2 &= x - 2 + 2 \\
 7 &= x
\end{align*} \]

Two step equations:

\[ 13 = 4x + 5 \]
\[ -6 = 2p - 2 \]
\[ \begin{align*}
 13 - 5 &= 4x + 5 - 5 \\
 8 &= 4x
\end{align*} \]
\[ \begin{align*}
 -6 + 2 &= 2p - 2 + 2 \\
 -4 &= 2p
\end{align*} \]
\[ \begin{align*}
 2 &= x
\end{align*} \]
\[ \begin{align*}
 -2 &= p
\end{align*} \]
Sandy's Boat House rents canoes. The equation \( c = 0.15t + 2.50 \) gives the charge \( c \) in dollars for renting a canoe for \( t \) minutes.

**Getting Ready for Problem 2.3**

- Explain what the numbers in the equation \( c = 0.15t + 2.50 \) tell you about the situation.
- Rashida and Serena apply for jobs at Sandy's. The manager tests them with three questions.

*What is the charge for renting a canoe for 30 minutes?*
*A customer is charged $8.50. How long did he use the canoe?*
*A customer has $10 to spend. How long can she use a canoe?*

Suppose you were applying for a job at Sandy's. How would you answer these questions?
A. For the first question, find the point on the graph with $x$-coordinate 30. The point is $(30, 7)$. This means that renting a canoe for 30 min costs $7. For the second question, find the point with $y$-coordinate 8.5. The point is $(40, 8.5)$. This means that for $8.50, you could rent a canoe for 40 min. For the third question, first find the point with a $y$-coordinate of 10. The point is $(50, 10)$. This means that for exactly $10, you can rent a canoe for 50 min. The customer can rent the canoe for any time of 50 min or less.
B. Rashida could use a table instead of a graph. Explain how to use a table to estimate answers to the questions.

B. She could make a table of \((time, charge)\) values for 5-min intervals. To answer the three questions, she would need to find the charge value corresponding to the time value 30 and the time values corresponding to the charge value $8.50 and $10. She could make the table with a graphing calculator by entering \(y = 0.15x + 2.50\) and setting the \(x\)-interval to 1.
C. Serena wants to find exact answers, not estimates. For the second question, she solves the linear equation $0.15t + 2.50 = 8.50$. She reasons as follows:

- If $0.15t + 2.50 = 8.50$, then $0.15t = 6.00$.
- If $0.15t = 6.00$, then $t = 40$.
- I check my answer by substituting 40 for $t$: $0.15(40) + 2.50 = 8.50$

Is Serena correct? How do you know?

C. Yes; she is applying the same operation to both sides of the equation at each step, so the sides remain equal. In the first step, she subtracts 2.50 from each side, or uses the related equation $0.15t = 8.50 - 2.50$. In the second step, she divides both sides by 0.15, or uses the related equation $t = 6.00 \div 0.15$, to get the solution. In the third step, she checks her solution by substituting it into the original equation and making sure the sides of the equation are equal.
For the third question, Rashida says, "She can use the canoe for 50 minutes if she has $10." Serena says there are other possibilities—for example, 45 minutes or 30 minutes. She says you can answer the question by solving the inequality \( 0.15t + 2.50 \leq 10 \). This inequality represents the times for which the rental charge is at most $10.

1. Use a table, a graph, and the equation \( 0.15t + 2.50 = 10 \) to find all of the times for which the inequality is true.

2. Express the solution as an inequality.

1. The charge will be $10 or less for any time from 0 minutes to 50 minutes. This can be seen on a graph of \( y = 0.15x + 2.50 \) by looking for points on this line that are also on or below the line \( y = 10 \). It can be seen in a table for \( y = 0.15x + 2.50 \) by looking for entries with cost values (\( y \)-values) less than or equal to 10. To use the equation, solve \( 10 = 0.15t + 2.50 \), which gives \( t = 50 \); then test \( t \)-values less than 50 in the inequality to see that the solution is \( t \leq 50 \).

2. The solution is all the \( t \)-values less than or equal to 50. This can be represented as \( t \leq 50 \). (Note that the model \( 1.5t + 2.50 \leq 50 \) does not match the context for all values of \( t \) because time is nonnegative.)
E. River Fun Paddle Boats competes with Sandy’s. The equation $c = 4 + 0.10t$ gives the charge in dollars $c$ for renting a paddle boat for $t$ minutes.

1. A customer at River Fun is charged $9. How long did the customer use a paddle boat? Explain.

2. Suppose you want to spend $12 at most. How long could you use a paddle boat? Explain.

3. What is the charge to rent a paddle boat for 20 minutes? Explain.

E. 1. 50 min; the solution of $9 = 4 + 0.10t$ is $t = 50$.

2. At most, 80 min; the solution of $4 + 0.10t \leq 12$ is $t \leq 80$.

3. $6; 4 + 0.10(20) = 6$

---

Solving for y-intercept:

M = 2; (3, 2)  \[ \begin{array}{cc} x & y \\ 2 & = 2(3) + b \end{array} \]

m = -3; (1, 4) \[ \begin{array}{cc} x & y \\ 4 & = -3(1) + b \end{array} \]

\[ \begin{array}{cc} 2 & = 6 + b \\ -6 & -6 \end{array} \]

\[ \begin{array}{cc} 4 & = -3 + b \\ +3 & +3 \end{array} \]

\[ -4 = b \]

\[ 7 = b \]
Inv 2.4 Intersecting Linear Models

A. Use the table to find a linear equation relating the probability of rain 
   $p$ to
   1. Saturday attendance $A_B$ at Big Fun.
   2. Saturday attendance $A_G$ at Get Reel.

   A. 1. $A_B \approx 1,000 - 7.5p$
        2. $A_G \approx 300 + 2p$

B. Use your equations from Question A to answer these questions. Show 
   your calculations and explain your reasoning.
   1. Suppose there is a 50% probability of rain this Saturday. What is the 
      expected attendance at each attraction?
   2. Suppose 460 people visited Big Fun one Saturday. Estimate the 
      probability of rain on that day.
   3. What probability of rain would give a predicted Saturday 
      attendance of at least 360 people at Get Reel?
   4. Is there a probability of rain for which the predicted attendance is 
      the same at both attractions? Explain.
B. 1. $A_B = 625$ people and $A_G = 400$ people
2. 72%
3. Solve $300 + 2p \geq 360$, which gives $p \geq 30$.
   So, a probability of rain of at least 30% will lead to movie attendance of at least 360.
4. Solve $1,000 - 7.5p = 300 + 2p$, or graph both equations and find the intersection point. This occurs when $p \approx 74$, so, for a probability of rain of about 74%, the predicted attendance is the same at both attractions.